Cross-Country Correlations in Sovereign Spreads

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Abstract

Over the last 30 years, the correlation across emerging market countries' sovereign debt spreads is more than double the correlation in their GDP (0.67 vs. 0.33). This discrepancy suggests that movement in sovereign spreads is primarily driven by global factors, not local fundamentals. Using data for 38 emerging market countries, I confirm that global variables are far more significant — have more than an order of magnitude larger R^2 — than local variables in explaining spread movement. Further, as evidence of the importance of price of risk channels for explaining spread movement, the share of a country's debt that is held by foreign investors significantly predicts the sensitivity of the spread to global financial conditions. I then build a three-period multi-country sovereign default model. The model shows that "standard" model features alone only produce spread correlations between 0.3-0.4. Introducing *either* cyclical investor risk-aversion *or* cross-country connections in variable costs of default matches the empirical correlation of 0.67. Yet, spreads in the model remain more tightly linked to fundamentals than in the data.

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1 Introduction

Movements in sovereign debt spreads are highly correlated across countries; far more so than movement in country fundamentals, like GDP. When Colombia's government finds it expensive to borrow, it tends to be the case that Indonesia's government also finds it expensive to borrow. Among 38 emerging market (EM) countries, the median cross-country correlation in spreads is 0.67. Meanwhile, the median cross-country correlations in detrended debt-to-GDP and GDP itself are 0.26 and 0.33. Sovereign spreads are seemingly yet another instance of asset price disconnect from fundamentals, as in the "excess volatility" puzzle of Shiller (1981).

What could explain such high correlation in spreads? Analytically I show that, in any binary choice default model where the Euler equation holds, for two generic countries A and B issuing risky debt, debt price correlations are determined by the ratio of a covariance and a variance. The covariance is between two objects: 1) how country A's default prospects are expected to move with the investor stochastic discount factor (SDF). 2) how country B's default prospects are expected to move with the investor SDF. The variance is how variable either object 1) or 2) is. Therefore, high spread co-movement could be generated by a variable SDF and relatively invariant default prospects for each EM country; or the converse, variable and linked default prospects which move similarly with respect to the SDF.

I extend the endogenous default model of Gilchrist et al. (2022) to a three-period two-country default model where a version of each of those two mechanisms can independently replicate a cross-EM spread correlation of 0.67. Mechanism 1) is cyclical investor risk-aversion creating variation in the price-of-risk. Mechanism 2) is fluctuating costs of default driving variation in the quantity-of-risk. Future empirical work is needed to establish the relative contribution of each of these channels. My decomposition of spread co-movement into "price-of-risk" and "quantity-of-risk" channels follows the work of Bai et al. (2023).

Across each model variety, however, spreads still co-move with country output far more than in the data. So, high spread correlations remain puzzling; and fit within the Mehra and Prescott (1985) tradition of asset pricing anomalies for macro models.

In addition to the model, I collect data on 38 EM countries' spreads over the last 30 years. I regress changes in the spread on a variety of local economy fundamentals and global financial variables. I find that *neither* set of variables does a good job explaining spread movement, but comparatively, global financial variables are much more informative. The

R^2 in regressions with only global variables is an order of magnitude larger than the R^2 in regressions with only local variables. The same pattern holds for changes in the cross-sectional standard deviation of spreads: global financial variables have roughly an order of magnitude more (in terms of R^2) explanatory power than domestic fundamentals. The idea that sovereign spreads are driven by common global factors is in keeping with Rey (2013)'s notion of the "Global Financial Cycle" (GFC).

I then provide a novel piece of evidence for the importance of global investors in driving spread movement: in a regression on changes in the spread, the interaction between the percent of a country's debt that is held by foreign investors and the change in the S&P 500 is significant, as is the interaction between the foreign-currency share of a country's debt and the change in the S&P 500. Both pieces of evidence come from Onen et al. (2023)'s newly released holdings data. Since exposure to foreign investors predicts the sensitivity of an EM's spread to the S&P 500, it must be that variation in the price of risk demanded by global investors contributes significantly to spread movement.

The fact that global financial variables significantly explain *some* of spread movement is motivation for introducing cyclical investor risk-aversion to the model. Cyclical risk-aversion is a reduced-form way of representing a variety of different "variable investor discount factor" micro-foundations.

The fact that spread movements are on the whole still poorly explained in my regressions is motivation for introducing stochastic and cross-country linked default costs. Arellano, Bai, and Lizarazo (2017) provide empirical evidence for cross-country default cost connections, and explicitly model a bargaining mechanism that links post-default renegotiation prospects across countries. My model again treats things in a reduced-form manner, with correlated exogenous shocks to the costs of default, leaving open different possible stories for what causes default cost co-movement.

Taken in combination, the results of the modeling and empirical exercises suggest that both variable risk-taking and linked default costs likely contribute to explaining high spread correlations. My use of reduced-form mechanisms sacrifices providing a specific account of what causes spread co-movement, in exchange for greater generality. The puzzle that lingers is how to specify a micro-founded story for either of these channels that does not end up linking spreads and EM output too closely together.

Outline: in section 2 I review the literature and explain in what sense high spread correlations are a "puzzle" for models. Section 3 presents summary statistics and regressions. Section 4 analyzes a general expression for spread correlations. Section 5 describes the model.

Section 6 gives numerical results. Section 7 concludes.

2 Position in the Literature

This thesis is primarily a contribution to the literature on sovereign default, and specifically, "endogenous default" models. "Endogenous default" refers to models where the decision to default is based on whether the benefit of repaying the debt outweighs the cost of default; not whether the country has the *ability* to repay their debt. Eaton and Gersovitz (1981) initiated the sovereign default literature. Aguiar and Gopinath (2006) and Arellano (2008) developed the modern, computational side of these models.

Most models in this tradition are single country models. The first contribution of my thesis is to make clear that single-country sovereign default models ignore one of the most salient features of EM sovereign spreads in the data: their high degree of international co-movement. Previous empirical work has noted this feature, i.e. Longstaff et al. (2011), Borri and Verdelhan (2011), and Aguiar et al. (2016); my model makes clear that matching high spread correlations is an issue for standard default models. Simply adding multiple countries and common risk-averse investors to a default model with "standard" features is insufficient.

The most similar extant paper to mine is Bai et al. (2023). They also emphasize the international dimension of spread movement, and build a model with additional sources of price of risk and quantity of risk movement in order to match the data. My model has two significant differences from theirs.

Firstly, the quantity of risk channel I model is linkages in the cost of default, while their mechanism is Bansal and Yaron (2004) style long-run risks. My use of cross-country default cost correlations is supported by the evidence and model in Arellano, Bai, and Lizarazo (2017). Arellano, Bai, and Lizarazo show that historically, lenders do recover a smaller fraction of defaulted debt when multiple countries are in default. Therefore, cross-country default cost linkages are more directly supported by the data than long-run risks, which Chen et al. (2022) show struggle with external validity.

Secondly, my model introduces price of risk variation by adding cyclical shocks to investor risk-aversion, while Bai et al. use Epstein-Zin preferences. Shocks to investor risk-aversion are less micro-founded than Epstein-Zin preferences, but more general. Risk-aversion shocks could reflect habits in consumption, or time-varying risk-bearing capacity of global financial intermediaries, like in the default model of Morelli, Ottonello, and Perez (2022). Risk-aversion shocks also avoid the issue with Epstein-Zin preferences pointed out by Epstein et al. (2014), that people would pay implausibly large amounts to resolve long-run risks that cannot affect their decisions.

There are two other theoretical contributions of my work worth highlighting: 1) I provide a general expression for the debt price correlation between generic issuers of risky debt. The only other analytical expression for this correlation I am aware of is in Tourre (2017), but he works in a more specific continuous time setting with a two-state Markov process for the investor SDF. 2) I add to the set of existing multi-country sovereign default models. The details of my model are most similar to (and were built from) the environment in Gilchrist et al. (2022), which builds closely off of Shin (2012). The crucial difference between our models is that in their model investors diversify away idiosyncratic sovereign risk, so there is no relevant notion of cross-country spread correlation — all country spreads behave the same in equilibrium. My setting is a flexible way of having multiple non-strategic countries present in a sovereign default model. As a three-period model, it is computationally much simpler than other multi-country default models, making it easier to evaluate the effects of different model features. Other important contributions to the multi-country sovereign default literature are Park (2014), de Ferra and Malucci (2020), and de Ferra and Romei (2023).

Another issue for default models, which my model is afflicted with as well, is that model spreads are tightly linked with country fundamentals; in particular, output and debt-to-GDP. My empirics show that this model feature is contradicted by the data. A regression of changes in the spread on changes in GDP, debt-to-GDP, and other country-level variables has an a R^2 of only 0.01. In conjunction, my empirics and model demonstrate that matching high spread correlations without over-predicting spread-to-fundamental correlations is a key challenge for sovereign default models going forward.

My empirics differ from others in the literature by focusing on changes in the spread, as opposed to the level of the spread. It has not been recognized before that changes in the spread have almost no discernible relation to country-level fundamentals, and that while global financial variables hold some explanatory power, they still do not explain changes in the spread very well ($R^2 < 0.2$). I also show that exposure to foreign investors interacts significantly with the sensitivity of a country's spread to global financial conditions. This finding supports the empirical evidence in Morelli, Ottonello, and Perez (2022) on the importance of global financial intermediaries for sovereign debt prices. My empirical results fit within the Rey (2013) and Miranda-Agrippino and Rey (2020) literature on the GFC, which Gilchrist et al. (2022) explicitly evaluate in the context of sovereign debt prices. Through the lens of this literature, the co-movement in sovereign spreads is part and parcel of a larger story that helps explain *all* global risky asset movement.

The final relation to the literature worth mentioning is on the general importance of discount factor variability for reconciling otherwise "puzzling" financial moments in macro models. Cochrane (2011) provides an assessment.

3 Empirics

3.1 Data and Summary Statistics

From Global Financial Data, I collected quarterly data on 38 different EM economy's JP Morgan Emerging Market Bond Index (EMBI) spreads, which combine different maturity dollar denominated bonds from a given country into one spread. The composite maturity is typically 2-5 years. My data stretches from 1993-2022, but is unbalanced, with most country's data only available for a subset of that time period. In total, I have 2,255 observations of quarterly spreads. Quarterly data on country GDP growth, the S&P 500 total return index, and the S&P 500 P/E ratio were also downloaded from Global Financial Data.

I used the publicly available data in Al-Amine and Willems (2023) for a variety of country-specific quarterly series, including debt-to-GDP and growth in private sector credit-to-GDP. That data also includes the CBOE's VIX and the Wu-Xia measure of the "shadow" federal funds rate.

The Global Financial Cycle (GFC) measure of Miranda-Aggripino and Rey (2020) comes from their replication package. Data on the Excess Bond Premium (EBP) comes from the replication package of Bauer and Swanson (2022).

Table 1 presents summary statistics, mirroring the presentation in Aguiar et al. (2016). The columns are, in order, the list of countries; their mean spread level; the quarterly standard deviation of their spread; the quarterly standard deviation of the *change* in their spread; their mean debt-to-GDP ratio; and the number of quarterly EMBI spread observations.

The units for the spread, its standard deviation, and the standard deviation of the change in the spread are basis points. Debt-to-GDP is a percentage.

The "median" row at the bottom is the median *of the country averages*. The median of the country means is the closest thing to a "representative" EM country, which is what the model will be calibrated to match. I will refer to these objects as the "median" going forward, keeping in mind that it is the median of the country means.

Table 1: Summary Statistics

Country	Spread (bp)	SD(Spread)	SD(ΔSpread)	Debt:GDP (%)	Obs.
Argentina	1431	1631	702	61	116
Belize	1051	528	332	87	30
Brazil	493	375	200	72	114
Bulgaria	336	284	126	40	63
Chile	152	53	39	13	94
China	119	57	35	28	82
Colombia	309	183	103	42	98
Dominican Republic	535	338	233	34	51
Ecuador	1172	877	761	35	111
Egypt	259	181	90	86	49
El Salvador	339	134	105	52	49
Gabon	413	237	197	25	27
Ghana	559	285	249	36	27
Hungary	181	159	71	68	62
Indonesia	287	139	99	33	41
Iraq	550	182	170	73	33
Jamaica	602	186	178	145	27
Kazakhstan	425	272	197	11	29
Lebanon	412	185	94	160	65
Malaysia	179	139	110	44	71
Mexico	291	143	84	45	100
Morocco	358	197	130	64	46
Nigeria	1100	749	517	36	65
Pakistan	663	458	243	66	53
Panama	303	127	70	53	72
Peru	289	188	91	32	100
Philippines	340	164	115	60	65
Poland	182	90	66	47	66
Russia	745	1217	702	28	67
Serbia	409	209	172	44	36
South Africa	189	95	63	35	62
Sri Lanka	587	417	312	76	27
Trinidad And Tobago	414	222	194	18	13
Tunisia	173	107	75	49	37
Turkey	391	234	128	53	67
Ukraine	513	300	151	32	52
Uruguay	387	298	155	77	53
Vietnam	313	145	108	48	35
Median	389	192	129	46	58

The median spread in my sample is quite high — 3.89%. There is significant variation in average spread levels: Argentina's average is a whopping 14.31%, while China's average spread is only 1.19%. Venezuela was cut from my sample for being too much of an outlier, with a mean spread of 48.04% (across 116 observations!).

The standard deviation of the spread displays similar heterogeneity, with a median of 1.92 pp. The median standard deviation of the change in the spread is slightly lower, but still substantial, given that it is a quarterly measure, at 1.29 pp.

The median EM country has sovereign debt equal to 46% of its GDP. The number of quarterly observations for debt-to-GDP does not always match the number of spread observations, and while spreads are on external debt, debt-to-GDP combines external and domestic debt.

As Aguiar et al. emphasize, what is notable is how high debt and spread levels are, and how volatile spreads are. Investors demand a significant and time-varying premium when investing in EM sovereign debt, yet, EM country's still accumulate large quantities of debt.

The central concern of this thesis is the large degree of co-movement observed across country spreads. Accordingly, Figure 1 displays the cross-country correlation matrix for quarterly changes in the spread.[1]

Correlation values are almost uniformly positive, and oftentimes quite high. High correlations are a global phenomenon, not regional. For example, look at the Colombia row, third from the bottom of the figure. The spread correlations with each of Gabon, Ghana, Serbia, Kazakhstan, and Sri Lanka are all greater than 0.8.

The median cross-country correlation in the change in the spread is 0.67. In comparison, the median cross-country correlation in log and filtered GDP is 0.33, and the median cross-country correlation in log and filtered debt-to-GDP is 0.26.[2]

Figure 2 anticipates the results of my regression analysis, on the lack of relationship between fundamentals and spreads. The x-axis represents a pair of countries' correlation in quarterly log and HP-filtered GDP, while the y-axis plots the correlation of the change in the spread for the same country pair. The red line of best fit indicates the lack of relationship

[1] The Morocco—Trinidad and Tobago pairwise correlation box is gray because they do not share overlapping spread change observations.

[2] I report averages of HP-filtered and Hamilton (2018) filtered numbers, in order to avoid taking a stand on optimal filtering. Discrepancies between the two are generally small in my sample. Filtering is important for debt-to-GDP because it exhibits a significant time trend. I log and filter output in order to correspond with output in the (stationary) model. The median cross-country correlation in plain GDP growth is even lower — 0.14. This latter number is more similar to what is reported elsewhere in the literature, i.e. 16% in Bai et al. (2023). Filtering the spread does not change its moments as it displays no trend.

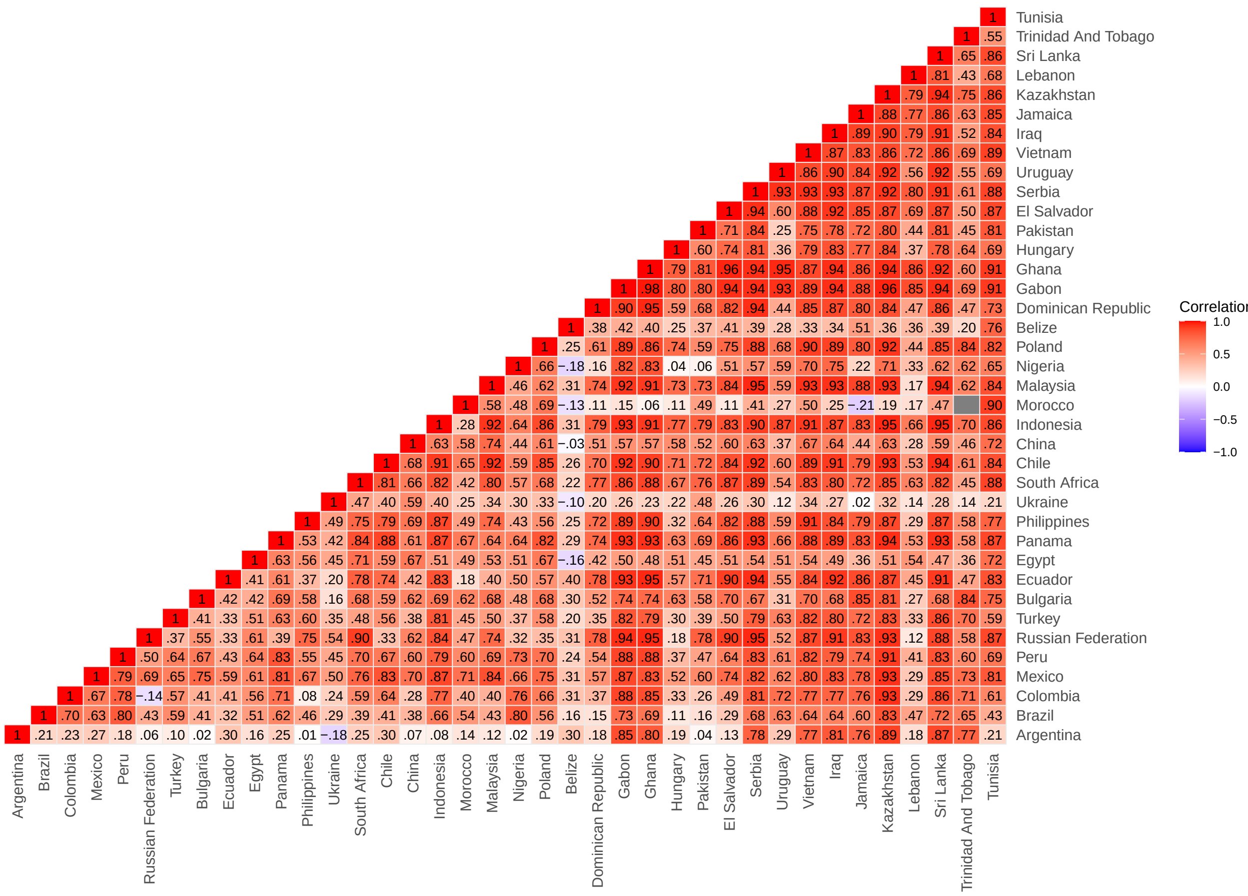

Figure 1: Cross-Country Correlation of Change in the Spread

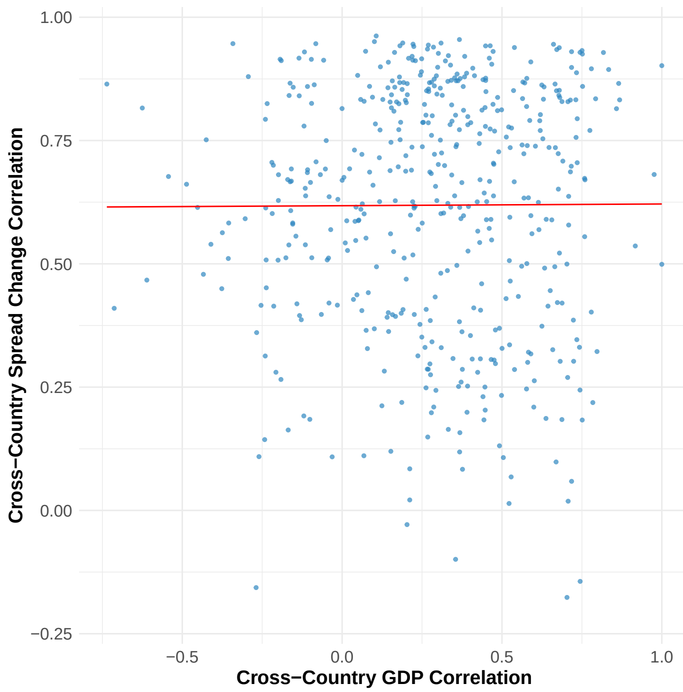

Figure 2: Cross-Country Correlation of GDP versus Cross-Country Correlation of Change in the Spread

between the two.

3.2 Regression Analysis

These regressions aim to establish two claims:

1) Explaining changes in spreads is challenging.

2) To the extent we can explain changes in spreads, global financial variables tell us much more than local fundamental variables.

3.2.1 Changes in Spreads

In order to better understand what drives spread movements, I run panel regressions of the following form:

$$\Delta S_{i,t} = \alpha_i + \beta_1 X_{1,i,t} + ... + \beta_n X_{n,i,t} + \epsilon_{i,t} \quad (1)$$

$\Delta S_{i,t}$ indicates the change in the spread of country i at time t; the intercept α_i is a country fixed effect; and $\{X_1, X_n\}$ is a set of n covariates. Regressions are estimated by stacking observations across countries and time, so β coefficients are the same for each country. Table 2 displays the first regressions of interest.

Before commenting upon the results, a note on interpretation: these regressions are not causally identified in any manner. When I use words like "explain" or "predict" to contrast the results, I am referring to statistical explanation and prediction. It very well may be that there are omitted variables which causally explain the associations in the regressions below. The assumption I am making when saying that these regressions provide evidence for the importance of global variables relative to local variables is that the omitted variable which could causally explain movement in *both* of (say) the S&P 500 and the EMBI spread must be some sort of "global" variable.

The first column is a "local only" regression, where only country-level independent variables are used. The regressors are the quarterly percent change in GDP, the year-on-year percent change in the real effective exchange rate (+ = appreciation), the year-on-year percent change in credit to the private sector as a percentage of GDP, the quarterly change in debt-to-GDP, the quarterly change in current account-to-GDP, and the quarterly change in the World Bank's governance index.

All variables have theory predicted signs, but only the exchange rate, credit, and debt-to-GDP variables are significant. The change in the spread is measured in basis points, so the coefficient on debt-to-GDP indicates that a ten percentage point increase in debt-to-GDP predicts a 36 basis point increase in that country's spread. The lack of significance of GDP growth is particularly puzzling for standard models of sovereign default.[3]

What stands out is the incredibly low R^2 — 0.01! — of the regression. Changes in local conditions explain virtually none of the variation in changes in spreads. The adjusted R^2 is even negative.

This total lack of explanation differs from the second column, where only global variables are used. The R^2 of 0.15 is still quite low, but is more than an order of magnitude larger.

The global variables used are the percentage change in the VIX; the percentage change in the GFC, which is a global factor that is higher when global financial conditions are better; the percentage point change in the shadow fed funds rate, the percentage change in the S&P 500 return index; the percentage change in the price-to-earnings ratio of the S&P 500; the

[3] The lack of significance remains if I use year-over-year GDP growth, HP-filtered growth, or if GDP growth is lagged across different horizons.

Table 2: Change in Spread Regressions

	Local Only	Global Only	Combined
ΔGDP (%)	−0.5		0.4
	(0.7)		(0.7)
ΔREER (%)	1.6*		0.6
	(0.8)		(0.7)
ΔCredit/GDP (%)	2.0**		1.2+
	(0.8)		(0.7)
ΔDebt/GDP	3.6*		5.4***
	(1.6)		(1.5)
ΔReserves/GDP	−3.4		−8.7*
	(4.4)		(4.1)
ΔCA/GDP	−4.9		−3.3
	(4.2)		(3.9)
ΔGov Index	−54.5		−53.6
	(149.5)		(138.8)
ΔVIX (%)		2.2***	2.3***
		(0.3)	(0.3)
ΔGFC (%)		0.0	0.0
		(0.0)	(0.0)
ΔSFFR		6.9	12.4
		(15.0)	(15.0)
ΔS&P (%)		−5.0***	−4.9***
		(1.0)	(1.0)
ΔP/E (%)		0.5+	0.6*
		(0.3)	(0.3)
ΔEBP (%)		0.0	0.0
		(0.0)	(0.0)
ΔUS GDP (%)		5.6	6.9+
		(3.6)	(3.6)
Num.Obs.	1430	1430	1430
R2	0.01	0.15	0.16
R2 Adj.	−0.01	0.13	0.14
BIC	19,793	19,580	19,608
RMSE	240.2	222.9	221.2

+ p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001

Regressions are $\Delta S_{i,t} = \boldsymbol{\beta X}_{i,t} + \epsilon_{i,t}$ with fixed effects for each country i. $\Delta S_{i,t}$ is the change in country $i's$ spread at time t. $\boldsymbol{X}_{i,t}$ is the vector of country i covariates at time t. GFC is a global financial cycle factor; SFFR is the Shadow Fed Funds rate; EBP is the excess bond premium. HC3 HC3 robust standard errors are in parentheses.

percentage change in the Excess Bond Premium (EBP), which measures the risk-premium on US corporate credit spreads, and is higher when global financial conditions are worse; and the quarterly percentage change in US GDP, converted to yearly terms.

Only the percentage change in the VIX and the percentage change in the S&P are significant at greater than 5% levels. As one would expect, increased volatility is associated with a rise in the spread, while high returns on the S&P correspond with lower spreads. The magnitudes also seem reasonable: a quarterly doubling of the VIX (100% increase) would predict a 220 basis point rise in spreads.

The final column includes all of the global and local variables from each individual regression. Importantly, adding local variables does not make any meaningful difference in predictability, relative to just having global variables. The adjusted R^2 increases by only 0.01, and the BIC increases when local variables are added, implying that on net adding local variables *hurts* model quality.

This pattern of results is not sensitive to the fact that I use independent variables that are contemporaneous with changes in the spread. Lagged variables are almost uniformly *less* significant. Since I am looking at a change in a financial variable, it would be surprising if lagged variables were predictive of future changes.

The *level* of the spread is much better explained by both local and financial variables, and the contrast between the explanatory power of the two is less stark.[4] Other empirical papers in the literature have focused on the spread level (i.e. Gilchrist et al. 2022). However, insofar as default models are meant to capture the dynamics of spreads, the change in the spread is the object to care about. The fact that changes in the spreads are almost *entirely* unexplained by local variables has not been sufficiently appreciated.

3.2.2 Changes in the Standard Deviation of Spreads

The next set of regressions I run examine quarterly changes in the cross-sectional standard deviation of spreads:

$$\Delta\,\mathrm{sd}(S_t) = \alpha + \beta_1 X_{1,t} + ... + \beta_n X_{n,t} + \epsilon_t \tag{2}$$

$\Delta\,\mathrm{sd}(S_t)$ is the change in the cross-sectional standard deviation of spreads, and all independent variables are cross-sectional variables or global variables. Even if country-level spread movement is better explained by global financial variables, you might think that changes in the dispersion of spreads are explained by changes in the dispersion of country

[4] Level regressions and regressions with lags are omitted from the appendix due to the word count limit.

fundamentals. Table 3 shows that is not the case: to the extent that spread dispersion is explained, global financial conditions do the explaining.

The three regressions are again a "local variable only" regression; a "global variable only" regression; and a combined regression. The "local variables" here are the change in cross-sectional average GDP growth; the change in cross-sectional average debt-to-GDP; the change in the standard deviation of GDP changes; and the change in the standard deviation of debt-to-GDP ratios. Cross-sectional averages are with respect to the EM countries whose spreads are part of the cross-sectional spread standard deviation at time t. All changes are quarterly changes.

Once again, the feature of the regression table to notice is the R^2. Cross-sectional local variables explain very little about changes in cross-sectional spread volatility ($R^2 = 0.03$), while global financial variables do explain a significant amount ($R^2 = 0.29$). Adding local variables to the global variable regression decreases model quality, as measured by the BIC.

The signs on individual variables do not tell as uniform of a story here, but some are readily interpretable: for example, based on the global only regression, a doubling of the VIX would predict a 210 basis point increase in the cross-sectional standard deviation of spreads. A one percentage point *fall* in the shadow fed-funds rate corresponds with a 77 basis point increase in the cross-sectional standard deviation of spreads. The change in the standard deviation is likely decreasing in the shadow fed funds rate because the rate was falling as global volatility spiked during the 2008 financial crisis.

3.3 Foreign Exposure Matters

There are two stories you could tell about the sensitivity of spreads to global financial variables: story 1) is a quantity of risk story. Perhaps global financial variables serve as good predictors of future fundamentals in EM countries, so EM spreads rise-and-fall due to the changes in their long-run economic prospects, and default likelihoods, augured by global financial conditions. Bai et al. (2023)'s use of long-run risks in their default model incorporates a version of this story.

Story 2) is a price of risk story. EM spreads move with global financial conditions simply because investors have time-varying desire and ability to bear-risk, even though actual default prospects are not changing. The evidence in Morelli, Ottonello, and Perez (2022) that sovereign bonds which happened to be held by banks more hurt by Lehman's collapse performed worse than bonds with otherwise identical characteristics supports this

Table 3: Change in Standard Deviation of Spread Regressions

	Local Only	Global Only	Combined
(Intercept)	−6.2	−38.2***	−37.0***
	(4.5)	(5.7)	(5.7)
ΔMean(ΔGDP)	−7.9***		2.1+
	(1.2)		(1.2)
ΔMean(Debt/GDP)	−3.2		4.0
	(2.2)		(2.6)
Δsd(ΔGDP)	−0.4		1.4***
	(0.4)		(0.4)
Δsd(Debt/GDP)	−2.9		−6.2+
	(3.5)		(3.4)
ΔVIX (%)		2.1***	2.3***
		(0.2)	(0.2)
ΔGFC (%)		0.01***	0.01***
		(0.00)	(0.00)
ΔSFFR		−77.3***	−72.2***
		(11.4)	(11.5)
ΔS&P (%)		−8.3***	−8.7***
		(0.7)	(0.8)
ΔP/E (%)		1.5***	1.4***
		(0.2)	(0.2)
ΔEBP (%)		0.0	0.0
		(0.0)	(0.0)
ΔUS GDP (%)		29.4***	29.7***
		(3.2)	(3.6)
Num.Obs.	2012	2012	2012
R2	0.03	0.29	0.30
BIC	27,061	26,464	26,468
RMSE	199.2	170.8	169.7

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Regressions are $\Delta\,\mathrm{sd}(S_t) = \boldsymbol{\beta}\boldsymbol{X}_t + \epsilon_t$. $\Delta\,\mathrm{sd}(S_t)$ is the quarterly change in the cross-sectional standard deviation of EM spreads. $\boldsymbol{X}_t$ is a vector of EM cross-sectional variables and global financial variables. GFC is a global financial cycle factor; SFFR is the Shadow Fed Funds rate; EBP is the excess bond premium. HC3 robust standard errors are in parentheses.

story.

In order to discriminate between the two, I merged my data on spreads with the newly released data in Onen et al. (2023) The combined data set has 691 quarterly spread observations. Their data reports the fraction of a country's combined domestic and foreign-currency debt that is held by foreign investors, and the fraction of a country's debt that is foreign-currency denominated. If denominating debt in foreign currency or having debt held by foreign investors makes a country's spread more sensitive to global financial conditions, that is evidence for story 2).

The two regressions I estimate in Table 4 are of the following form:

$$\Delta S_{i,t} = \alpha + \beta_1 \Delta \left(\text{S\&P}\right)_t + \beta_2 X^*_{i,t} + \beta_3 \Delta \left(\text{S\&P}\right)_t \times X^*_{i,t} + \beta_4 \, \text{Cor}_i + \epsilon_t \tag{3}$$

$\Delta S_{i,t}$ is still the quarterly change in the spread. For ease of interpretation, both regressions have only three independent variables. Those three variables are 1) the quarterly percentage change in the S&P 500 2) X^*_t, which is a "foreign investor exposure" variable. In one regression, it is the percentage of an EM's debt held by foreign investors. In the other, it is the percentage of an EM's debt which is denominated in foreign currency. 3) The correlation between changes in that EM's output and changes in US output, denoted Cor_i.

The coefficient of interest is β_3, on the interaction between changes in the S&P and the foreign investor exposure variable. If it is significant, that implies that having a greater proportion of overall debt held by foreign investors makes the spread on USD denominated debt more sensitive to US financial conditions. That is, the more a countries debt is held by foreigners, the more its spread is determined by fluctuations in the price of risk.

The omitted variable which threatens this interpretation is if the percent of debt held by foreign investors is increasing in how much an EM's default prospects move with US financial conditions. That threat to exogeneity would not make sense from a US investors perspective, since investors would be holding more of the debt which provides less of a hedge, but could arise due to frictions in financial markets.

I include Cor_i to address this concern. If the interaction term remains significant when controlling for the correlation between a countries output and US output, that suggests the price-of-risk channel is at work.

Table 4 shows that, indeed, even when controlling for a countries output correlation with the US, the interaction between US S&P returns and foreign investor exposure remains significant. The first column reports the results where the share of debt held by foreign

Table 4: Foreign Investor Exposure on Change in the Spread Regressions

	Foreign Investor Share	Domestic Currency Share
(Intercept)	−6.9	80.2+
	(16.0)	(48.3)
ΔS&P (%)	−2.4	−31.9***
	(1.7)	(9.6)
Foreign Investor Share (%)	0.9*	
	(0.5)	
Cor_i	4.0	17.3
	(17.8)	(18.0)
ΔS&P × Foreign Investor Share	−0.19*	
	(0.09)	
Domestic Currency Share (%)		−0.9
		(0.6)
ΔS&P × Domestic Currency Share		0.32**
		(0.10)
Num.Obs.	691	691
R2	0.25	0.33
RMSE	126.1	118.9

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The dependent variable is $\Delta S_{i,t}$, the change in country $i's$ spread at time t. In both regressions, the coefficient of interest is the interaction term. HC3 robust standard errors are in parentheses.

investors is the exposure term, while the second column is the share of debt issued in foreign currency.

The interaction between S&P returns and foreign investor holding share is significant at the 5% level. The negative coefficient indicates that the greater the fraction of an EM country's debt is held by foreign investors, the more that countries spread *falls* as the S&P rises.

In column two, the sign is flipped, since the exposure variable is now the percentage of an EM country's debt that is denominated in domestic currency. The positive coefficient, significant at the 1% level, indicates that the greater the fraction of an EM country's debt is issued in domestic currency, the *less* that countries spread falls as the S&P rises. The coefficient on the S&P return alone is negative in both regressions, and significantly so in the second one. On its own, as the GFC story would predict, positive returns on the S&P predict lower spreads. But the interaction term shows that this effect is lessened if a country issues a greater proportion of their debt in domestic currency.

The significance of these interaction terms remains if I add in the host of local and global co-variates used in my previous regressions. If the foreign investor share or domestic currency share is interacted with the other global financial variables in the regression, it also tends to be significant, though not in every single iteration.[5] The significance of the interaction term does not determine *how* important the price of risk story is relative to the quantity of risk story in explaining spread movement; but it does suggest that spread movements are at least in part explained by fluctuations in the price of risk, and that those fluctuations stem from global financial conditions.

4 Stylized Theory

I begin with a general presentation of the theoretical cross-country correlation in sovereign debt prices. As will be seen, the correlation in debt prices is quite a non-linear object, making it challenging to derive analytically interpretable expressions for. Tourre (2017) conducts a similar exercise, but his expression differs from mine since he operates in a continuous time setting with a two-state investor SDF. The results here are meant to fix ideas about what could, in theory, drive cross-country correlation in spreads.

[5] Omitted from the appendix due to the word limit.

In almost all sovereign default models, the price of sovereign debt can be written:

$$q_t^i = \mathbb{E}_t\left[m_{t+1}(1-D_{t+1}^i)\right] \tag{4}$$

q_t^i represents the price of country i's debt at time t; m_{t+1} is the investors stochastic discount factor (SDF) at time $t+1$; and D_{t+1}^i is country i's default decision at time $t+1$. In sovereign default models, $D^i \in \{0,1\}$ to reflect that defaulting or not is a binary decision. The spread on country i's debt will be given by $1/q^i - (1+r_f)$, where r_f is the risk-free net interest rate.

Suppose there is some generic shock Z which occurs between periods t and $t+1$. When the shock occurs, the price updates to reflect new expectations about investors' SDF and country default prospects. Holding fixed the risk-free rate, the pre-shock expected cross-country correlation in spreads upon the shock will move with the expected cross-country correlation in prices, which for an abstract country 1 and country 2 is given by:

$$\text{Cor}(q^1, q^2) = \frac{\mathbb{E}_t\left[(q^1|Z)(q^2|Z)\right] - \mathbb{E}_t[q^1|Z]\,\mathbb{E}_t[q^2|Z]}{\sqrt{\text{Var}(q^1|Z)\,\text{Var}(q^2|Z)}}$$

The expectations here are taken before the shock occurs, so $\mathbb{E}\big(q^i|Z\big)$ and $\text{Var}\big(q^i|Z\big)$ are themselves random variables. In order to take expectations, I assume that Z has a distribution with a defined first moment and that its support is the real line.

Now, further assume that, before the shock occurs, each country is generic. The expected variance of each country's price of debt is equal, and the expected co-movement between the product of the country's default prospects with the SDF and the investors SDF is the same for each country. This is meant to be a benchmark, where each country is a "representative" EM economy. Formally:

$$\text{Var}\big(q^1|Z\big) = \text{Var}\big(q^2|Z\big) \text{ and } \text{Cov}\Big(\mathbb{E}\big(mD^1|Z\big), \mathbb{E}(m|Z)\Big) = \text{Cov}\Big(\mathbb{E}\big(mD^2|Z\big), \mathbb{E}(m|Z)\Big) \tag{5}$$

This is not saying that for all specific shock realizations $z \in Z$ the co-movement of default prospects and the SDF is the same for each country, but that, in expectation, they are the same. Each country is "generic" with respect to investors' SDF. As shown in appendix A1, using D^i for terms that are equivalent across country 1 and 2, the correlation expression is now:

$$\begin{aligned}\text{Cor}(q^1, q^2) &= \frac{\text{Cov}(\text{E}[mD^1|Z], \text{E}[mD^2|Z]) - 2\text{Cov}(\text{E}[mD^i|Z], \text{E}[m|Z]) + \text{Var}\,(\text{E}[m|Z])}{\text{Var}\,(\text{E}[mD^i|Z]) - 2\text{Cov}(\text{E}[mD^i|Z], \text{E}[m|Z]) + \text{Var}\,(\text{E}[m|Z])} \\ &= \frac{\text{Cov}(\text{E}[mD^1|Z], \text{E}[mD^2|Z]) - X}{\text{Var}\,(\text{E}[mD^i|Z]) - X}\end{aligned} \tag{6}$$

The correlation is a highly non-linear object. It does not *just* depend on the covariance of default prospects with the SDF, or the cross-country covariance of default prospects. Rather, it depends on terms like the covariance of each country's expectation of the product of the investor SDF and country default prospects.

The only difference between the numerator and denominator are the leftmost terms. The second line indicates this by replacing the terms that are identical on top and bottom with X. So, what leads to the correlation being less than one is *solely* the discrepancy between $\text{Cov}(\text{E}[mD^1|Z], \text{E}[mD^2|Z])$ and $\text{Var}\left(\text{E}[mD^i|Z]\right)$. Cross-country spreads are highly correlated when the co-movement between the expected product of the SDF and country 1 default prospects and the product of the SDF and country 2 default prospects is similar to the variance of the product of the SDF and country default prospects (for either country).

It is a mouthful, but it makes sense: when each country's default prospects moves with the investor SDF *in the same way*, that is when the correlation is high. It is not enough for default prospects alone to move together — default prospects must move similarly *with* the SDF. Therefore, a high correlation could arise from the SDF being relatively stable while default prospects move together, *or* from default prospects being relatively invariant while the SDF moves a lot.

If I didn't make the assumptions in (5), the correlation expression would be more complicated. However, those assumptions are only that each country behaves similarly with respect to marginal investors, not that they always behave similarly to each other. In the context of global investors investing in a diversified portfolio of EM countries, this assumption seems reasonable. As long as two countries are mostly similar with respect to marginal investors, the primary determinant of their spread correlation will be expression (6).

A partial equilibrium exercise that helps build some intuition is taking the derivative of the correlation (6) with respect to $\text{Var}(\mathbb{E}(m|Z))$, while holding all other terms fixed. I denote $\text{Cor}\left(q^1, q^2\right)$ as $\rho_{1,2}$, giving that:

$$\frac{\partial \rho_{1,2}}{\partial\, \text{Var}(\mathbb{E}(m|Z))} = \frac{1 - \rho_{1,2}}{\text{Var}(q^i|Z)} \tag{7}$$

When the expected investor SDF becomes more variable, the correlation between debt prices increases more when 1) the smaller is the correlation to begin with (the larger is $1-\rho_{1,2}$) and 2) the smaller is the variance of prices to begin with. If there is not much variability in default prospects, so the variance of prices are small, then a given increase in the variability of investors' SDF has a bigger impact on the correlation in prices.

The takeaway from this exercise is not shocking, but it is important to keep in mind: co-movement in default prospects or variability in the SDF alone do not guarantee a high cross-country correlation of debt prices (and spreads). It is the joint movement of default prospects and the SDF, *across countries*, that determines cross-country spread correlations.

5 The Model

The model builds off the endogenous default model in Gilchrist et al. (2022), itself built off of Shin (2012)'s model with exogenous default. The primary differences are the introduction of a third-period, dropping the assumption that investors diversify away idiosyncratic country risk, dropping global banks from the model, and adding investor outside income. The first two assumptions are necessary to be able to examine the correlation in spreads.

5.1 Borrowing Countries

There are three periods of time $t = 0, 1, 2$. Income is fixed at zero in period zero and stochastic in period one and two. An emerging market (EM) country i consumes only in periods zero and two, and maximizes lifetime utility over those two-periods:

$$\max_{b^i} \left\{ \frac{(c_0^i)^{1-\gamma}}{1-\gamma} + \beta\, \mathbb{E}_0 \left[\frac{(c_2^i)^{1-\gamma}}{1-\gamma} \right] \right\} \tag{8}$$

$$\text{s.t. } c_0^i = q_0^i b^i \tag{9}$$

$$c_2^i = \max \left\{ y_2^i - b^i, (1 - \phi_1 e^{\phi_2 z_2^i}) y_2^i \right\} \tag{10}$$

Since the sovereign receives no income in period zero, they are incentivized to borrow b^i to smooth consumption.[6] Income in period two is randomly given by $y_2^i = e^{z_2^i}$, where $z_2^i = k_{EM} Y_2 + x_2^i$. Here, Y is a global income factor, x^i a country-specific idiosyncratic

[6] This assumption could be replaced with assuming sovereigns have a very low discount factor β.

income factor, and k_{EM} is the loading of EM countries on the global income factor. The income factors evolve according to:

$$Y_2 = \rho_Y Y_1 + \epsilon_{Y,2}, \text{ where } \epsilon_{Y,2} \sim N(0, \sigma_Y^2) \tag{11}$$

$$x_2^i = \rho_x x_1^i + \epsilon_{x^i,2}, \text{ where } \epsilon_{x^i,2} \sim N(0, \sigma_x^2) \tag{12}$$

Each factor starts off in period zero at its long run mean of 0, takes a realization in period one given by $\epsilon_{Y,1}, \epsilon_{x^i,1}$ (with the same distribution as time-two shocks), and then is determined in period two by the above equations.

Consumption in period two is a choice between consuming output minus the debt that was borrowed in period zero, or consuming some fraction of output if the sovereign chooses to default. Default is indicated by $D^i = 1$. Parameters ϕ_1 and ϕ_2 govern, respectively, the level and curvature of the non-linear default cost function.

At time zero, the sovereign chooses borrowing to maximize expected lifetime utility. At time two, the sovereign chooses whether or not to default — $D^i \in \{0, 1\}$.

The intervening shock at $t = 1$ should be thought of as representing any information that comes out between borrowing and repayment. The information causes debt prices to update on secondary markets from q_0 to q_1, but does not affect the sovereign's problem. I calibrate the model to quarterly data, so debt is of two-quarter maturity, but in principle, this structure could represent any maturity debt, with the period one shock representing any information realized between issuance and repayment.

The sovereign is indifferent between defaulting and repaying when $e^{z_2^i} - b^i = (1 - \phi_1 e^{\phi_2 z_2^i}) e^{z_2^i}$, which, as shown in appendix A2, results in a time zero default probability p^i of:

$$p^i = \Phi\left(\frac{\ln\left(\frac{b^i}{\phi_1}\right)}{\sigma_z(1 + \phi_2)}\right) \tag{13}$$

Φ represents the standard normal CDF. The probability of default is increasing in the amount borrowed, decreasing in the level of default costs ϕ_1, increasing in the curvature of default costs ϕ_2, and increasing in the variance of income σ_z^2. From the standpoint of time zero, default decision D^i is a Bernoulli random variable with $\mathbb{E}(D^i) = p^i$ and $\text{Var}(D^i) = p^i(1 - p^i)$.

5.2 Investors

Global investors have mean-variance preferences and must choose what fraction of their one unit of wealth to allocate to sovereign debt from two issuing countries:

$$\max_{a^1,a^2} \quad \mathbb{E}(W) - \frac{\gamma^w}{2}\,\mathrm{Var}(W) \tag{14}$$

Holdings of country 1 and 2 debt are a^1 and a^2. γ^w is investor risk-aversion, possibly different from sovereign risk-aversion, where lowercase w stands for "world." Capital W is period 2 wealth.

Wealth not invested in sovereign debt earns the risk-free rate of return. Investors have period 2 outside income $y_2^w = e^{z_2^w}$, where $z_2^w = k_w Y_2 + w_2$. The global income factor remains the same, but investors loading k_w differs, and idiosyncratic income w_2 has its own variance σ_w^2.

The expectation and variance of investor wealth in period 2 are:

$$\mathbb{E}(W) = (1 - a^1 - a^2)(1 + r_f) + \frac{a^1}{q^1}\left(1 - \mathbb{E}\big(D^1\big)\right) + \frac{a^2}{q^2}\left(1 - \mathbb{E}\big(D^2\big)\right) + \mathbb{E}(y_2^w) \tag{15}$$

$$\begin{aligned}\mathrm{Var}(W) = &\left(\frac{a^1}{q^1}\right)^2 \mathrm{Var}\big(D^1\big) + \left(\frac{a^2}{q^2}\right)^2 \mathrm{Var}\big(D^2\big) + \mathrm{Var}(y_2^w) \ + \\ 2&\left(\frac{a^1 a^2}{q^1 q^2}\,\mathrm{Cov}\big(D^1, D^2\big) - \frac{a^1}{q^1}\,\mathrm{Cov}\big(D^1, y_2^w\big) - \frac{a^2}{q^2}\,\mathrm{Cov}\big(D^2, y_2^w\big)\right)\end{aligned} \tag{16}$$

Investor demand a^1 for country 1 debt is then given by maximizing (14) with respect to (15) and (16):

$$a^1 = \frac{q^1}{\mathrm{Var}(D^1)}\left(\frac{1 - \mathbb{E}\big(D^1\big) - (1 + r_f)q^1}{\gamma^w} - \frac{a^2}{q^2}\mathrm{Cov}(D^1, D^2) + \mathrm{Cov}\big(D^1, y_2^w\big)\right) \tag{17}$$

A symmetric condition holds for a^2.

There exist N investors, so total demand for sovereign one's debt is given by Na^1. Market clearing implies that $b^1 = Na^1$ in equilibrium.

Everything done thus far has been in terms of two countries. Two countries is sufficient to have a relevant notion of cross-country spread correlation, and keeps things simpler, but the investor problem can easily be extended to any number of countries. Changing the number of investors N will scale up the total amount of investor resources, leaving the relevant economics unchanged.

5.3 Equilibrium

The time-zero equilibrium is a set of prices $\{q_0^1, q_0^2\}$, borrowing by sovereigns $\{b^1, b^2\}$, and investor asset demands $\{a_0^1, a_0^2\}$ which satisfy market clearing: $b^1 = Na_0^1$ and $b^2 = Na_0^2$. Prices must be such that borrowing and asset demands satisfy the sovereign and investor maximization problems. The equilibrium is a rational expectations equilibrium so forward-looking expectations, variances, and covariances are accurate.

Equilibrium is defined and computed for the two-country case, but since the model dynamics remain unchanged for an arbitrary amount of countries, the countries are not strategic in their behavior. Each country solves its own maximization problem taking prices as given.

The model also has a time-one rational expectations equilibrium. Country borrowing decisions have already been made, so prices no longer need to satisfy their maximization problem. Instead, given period one shocks, prices $\{q_1^1, q_1^2\}$ must update so that investors still demand their previous levels of $\{a_1^1, a_1^2\}$. That is, secondary markets clear.

5.4 Time-Zero Investor Demand

At a time-zero equilibrium, the two EM's are ex-ante identical and they will borrow the same amount. Prices and default probabilities are the same for each country. I equate $a_0^1 = a_0^2$ in (17) to get equilibrium investor asset demand a_0^1 in terms of price q_0^1:

$$a_0^1 = \frac{q_0^1 \left(1 - \mathbb{E}\big(D^1\big) - (1 + r_f)q_0^1 + \gamma^w \operatorname{Cov}\big(D^1, y^w\big)\right)}{\gamma^w \left(\operatorname{Var}(D^1) + \operatorname{Cov}(D^1, D^2)\right)} \tag{18}$$

In appendix A.3 and A.4, I show that investor demand a_0^1 is decreasing in default probability p^1; decreasing in investor risk-aversion γ^w; decreasing in the risk-free rate; decreasing in the correlation of defaults across country one and two; and increasing in the covariance between investor income and country one default likelihood (a higher covariance implies debt is less risky when investor income is low).

Given the expression for default probabilities (13), it follows that investor demand for country one debt obeys the following comparative statics with respect to exogenous variables:

- increasing in the level of default costs ϕ_1 and decreasing in the curvature ϕ_2

- decreasing in country one's income variance σ_z^2, as well as each component σ_Y^2 and σ_x^2
- increasing in the EM loading on the global factor k_{EM}
- increasing in investor income loading on the global factor k_w

The same comparative statics hold for country two asset demand.

5.5 Time-One Investor Demand

The time one shock $\mathcal{Z} = \{\epsilon_{1,Y}, \epsilon_{1,x^1}, \epsilon_{1,x^2}, \epsilon_{1,x^w}\}$ updates all expectations, variances, and covariances of random variables. Equilibrium asset demand must remain the same for each countries debt, so for any movement in expectations and covariances that would induce demand changes, prices move inversely to maintain equilibrium.

Equilibrium asset demand $a_1^1 = a_0^1$ in terms of prices q_1^1, q_1^2 is now:

$$a_1^1 = \frac{q_1^1 q_1^2 \left(1 - \mathbb{E}\big(D^1|\mathcal{Z}\big) - (1+r_f)q_1^1 + \gamma^w \operatorname{Cov}\big(D^1, y^w|\mathcal{Z}\big)\right)}{\gamma^w \left(q_1^2 \operatorname{Var}(D^1|\mathcal{Z}) + q_1^1 \operatorname{Cov}(D^1, D^2|\mathcal{Z})\right)} \tag{19}$$

The correlation in spreads is determined by how much a_1^1 and a_1^2 *would* move together in the absence of price adjustments.

To get a feel for how things change at time-one, Figure 3 plots country 1 default probabilities $p^1 = \mathbb{E}\big(D^1|\mathcal{Z}\big)$ as a function of $\mathbb{E}\Big(e^{z_2^1}|\mathcal{Z}\Big)$ — country 1 expected period-two output conditional on time-one shocks. As can be seen, default probabilities are quite non-linear in expected period-two income.

The blue line uses the baseline parameter calibration and equilibrium debt level presented in Section 6.1. The orange line scales up the standard-deviation of one-quarter ahead income by 20%, while the green line scales it down by 20%.

Default probabilities (13) change in two ways upon the time time-one shock: one-quarter ahead income variance σ_z^2 is lower than two-quarter ahead income variance, which tends to push down default likelihood. On average, the resolution of uncertainty reduces the chances of default, producing an upward sloping term structure for spreads. The standard deviation of time-two output at time-zero is about twenty percent higher than at time-one. At the mean expected period two output value of 1, the gap between the orange line and the blue line represents the change in default probability caused by the resolution of uncertainty.

What changes in a different way upon each realization of the shock is expected period two output, for both sovereigns and for the investor. Low output realizations in period one

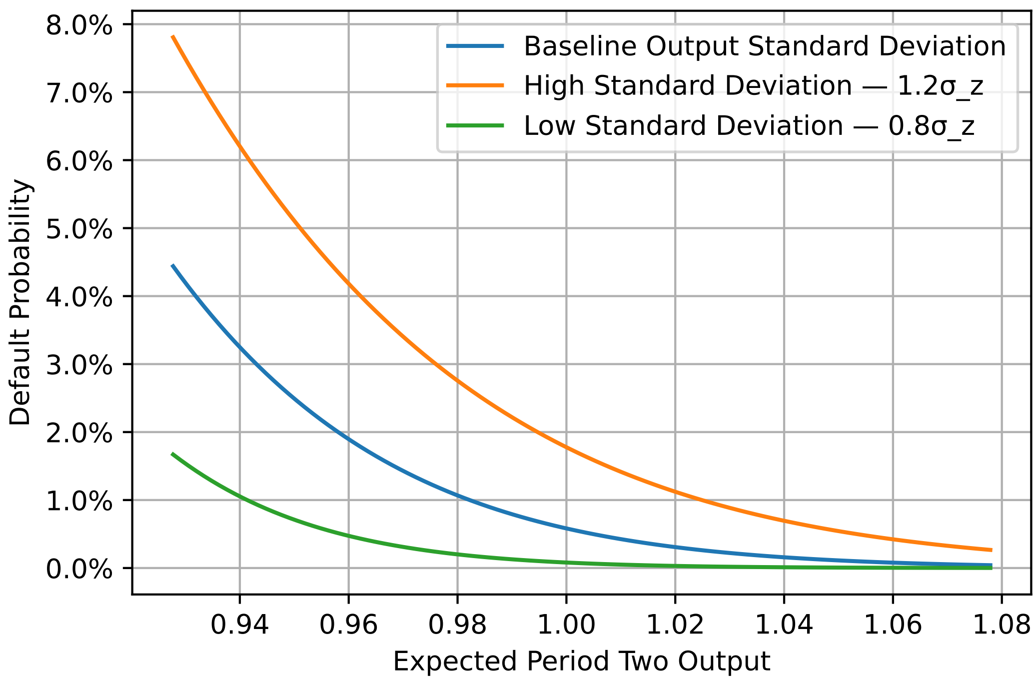

Figure 3: Default Probability and Expected Period-Two Output

reduced expected period two output, and non-linearly increase the probability of default in period two.

Some parameters have an unambiguous affect on the realized spread correlation. Increasing loadings on the global factor k_{EM} or k_w pushes up cross-EM and/or investor-EM output correlations, and likewise increases the spread correlation.

Other parameters have conflicting effects: increasing the auto-correlation of the idiosyncratic component of EM output ρ_x decreases the period-two cross-EM output correlation, but time-two expected output becomes more dependent on the time-one shock, and therefore more variable. Due to the non-linear relationship shown in Figure 3, greater variability of expected period two output causes more than one-for-one increases in the conditional variance and expectation of time-two default. As a result, even though cross-EM output correlation is lower, higher conditional default variances can cause $\mathrm{Cov}\left(D^1, D^2\right)$ to rise, and possibly lead to higher equilibrium spread correlations. In my calibration, it turns out that the relationship between ρ_x and the spread correlation is non-monotonic: decreasing for small increases in ρ_x but increasing for larger increases. The non-monotonicity occurs precisely because of the non-linear relationship between output variability and default probability variability.

There is nothing particularly important about the ρ_x parameter, but it illustrates the complexity of the cross-EM spread correlation. Even in a simple model structure, with no unusual features, the various non-linearities produce non-monotonic relationships between exogenous parameters and spread correlations.

Other comparative statics of interest for the spread correlation, verified numerically, are:

- The correlation is increasing in the variance of the global income factor σ_Y^2. Increasing the variance of the global income factor increases cross-EM and EM-investor output correlations, while making output (and default prospects) more variable.
- For the same reasons, the correlation is increasing in the auto-correlation of the global income factor ρ_Y.
- Increasing in the level of investor risk-aversion γ^w, which pushes up the sensitivity of each country's spread to the covariance of default prospects with investor income.
- Decreasing in the number of investors N. The more investors there are, the smaller a fraction of an individual investor's wealth is held in EM debt. Less wealth in EM debt reduces the impact of one country defaulting on an investors level of utility, and therefore reduces spillovers between worse default prospects in one country and the risk-premium investors demand on the other country's debt.

6 Numerical Results

Typically, two/three-period models are used to show analytic results, while infinite-horizon models are used to show quantitative results. Nonetheless, I use the time-one "shock" trick to make a number of quantitative points. I believe this is defensible insofar as what "really" determines the quantitative properties of sovereign default models is the default cost function. This is a drawback of endogenous default models — it offers too many degrees of freedom to the modeler, as argued by Guimares and Tumkus (2020) — but it is current reality.

In a model like mine, the default cost function in period two is not a static cost that only represents how much output is lost *in that period*. Rather, the default cost function

Table 5: Baseline Calibration

Fixed Parameter	Target
$\gamma^{EM} = 2$	Standard in Lit
$\gamma^{w} = 12$	Standard for *Intra*temporal
$\beta = 0.98$	
$r_f = 0.01$	Mean shadow fed-funds rate of 2%
$k_{EM} = 0.69$	EM cross-country output correlation of 0.325
$k_w = 0.64$	Median EM Output correlation w/ US of 0.435
$\sigma_Y, \sigma_x = 0.0235$	EM quarterly output std. of 0.0285
$\sigma_w = 0.0125$	US quarterly output std. of 0.0195
$\rho_Y, \rho_x = 0.68$	EM Quarterly output auto-correlation of 0.68
$\rho_w = 0.77$	US Quarterly output auto-correlation of 0.72
N = 9	10.3% of investor wealth in EM debt
Calibrated Parameter	
$\phi_1 = 0.58$	2% default probability, 46% B/Y, 3.89% spread
$\phi_2 = 2.2$	2% default probability, 46% B/Y, 3.89% spread

should represent the true "costs of default" relative to re-paying the debt. Those costs extend out into the future, and the default cost function represents them in units of period two utility relative to the utility of paying back the debt. Given that interpretation, and given the degrees of freedom allowed in default cost functions across the literature, there is no reason the quantitative realism of a two-period model should be significantly different from an infinite-horizon model. Of course, there are questions a two-period model cannot address — like default recoveries, or the dynamics of debt-to-output ratios. But many of the core moments and questions can be quantitatively analyzed. I conjecture that the pattern of results I find — about what can explain spread co-movement — will hold in an infinite-horizon model. Verifying that conjecture is left for future work.

Appendix A.5 provides details on how to solve the model.

6.1 Baseline Calibration

The parameters "fixed" by outside sources are at the top of Table 5, while the calibrated parameters are at the bottom.

EM and investor risk-aversion differ. EM risk-aversion is set at a standard value of 2. Investor risk-aversion is set to 12. Investor risk-aversion is "high" because the investor problem is static: γ^w is only an intra-temporal risk-aversion coefficient; not an intertemporal elasticity of substitution (IES). Therefore, it should correspond with risk-aversion coefficients in models with Epstein-Zin preferences, where risk-aversion is separated from the IES. The value of 12 is standard in the literature and matches the risk-aversion coefficient used in Bai et al. (2023)'s default model with Epstein-Zin preferences.

A time-zero risk-free rate of .01 is set to match an annual mean shadow fed funds rate of 2%, since the time-zero risk-free rate is a two-quarter long rate. I assume that the idiosyncratic and global components of EM income share the same standard deviation and auto-correlation, and that investor income follows the US output process. All targeted EM and US output moments are the *average* of the log and HP-filtered moments and the log and Hamilton (2018) filtered moments in my data. I average the two to avoid taking a stand on optimal filtering. The parameters are set so that the moments of the period-one income realizations match the data.

The number of investors N is set to 9 so that 10.3% of investor wealth is held in EM debt. 10.3% comes from taking the data in Fang, Hardy, and Lewis (2022) and multiplying the share of EM debt held by different investor types by the share of that types wealth that is held in EM debt. Appendix A.6 provides further details.

The two calibrated parameters determine the shape of the default cost function. They are picked to match three time-zero equilibrium static moments: 46% debt-to-GDP[7], 2% unconditional default probability, and a 3.89% spread. The debt-to-GDP and spread figures are the median of country means reported in Table 1. The 2% default probability target matches the target used in Bai et al., and splits the difference between a 3.3% default frequency in my data when using the Medas et al. (2018) definition of a fiscal crisis, and a 0.8% default frequency when using Erce, Mallucci, and Picarelli (2022)'s database.

6.2 Baseline Results

Table 6 presents the model's key static and dynamic moments across a few different specifications. Column 1 is the baseline calibration just presented. In column 2, investor risk-aversion is set to 2. Column 3 sets $k_w = 0$, making investor income uncorrelated with EM output.

[7]Debt-to-GDP in the model is the period two level of debt-to-GDP at the mean level of period two output, which is 1.

Table 6: Baseline Model Moments

Variable	Model			Target
	$\gamma^w = 12$	$\gamma^w = 2$	$k_w = 0$	
B/Y (%)	0.46	0.47	0.46	0.46
P(Default) (%)	1.85	3.50	2.24	2.00
Spread (%)	3.83	4.28	3.95	3.89
$\text{Cor}(\Delta S^1, \Delta S^2)$	0.36	0.31	0.33	0.67
$\text{sd}(\Delta S)$	0.65	1.00	0.93	1.29

The two dynamic moments I evaluate the model on are a cross-country correlation of the change in spreads of 0.67 and a standard deviation of the change in the spread of 1.29 pp. The change in spreads is the difference between the time-zero and time-one spread, where the risk-free rate halves to reflect that at time-one it is a one-quarter rate. The reported standard deviation is the average of each country's own standard deviation.

All three model specifications do a good job matching the static moments, but fail to replicate the dynamic moments.

The baseline model with $\gamma^w = 12$ only produces a 0.36 cross-country correlation in spreads, and the change in the spread has about half its standard deviation in the data. The change in the spread correlation is slightly higher than the cross-EM log output correlation (0.325), so there is some degree of spread co-movement "beyond" fundamentals, but it is very limited.

The next two columns show that if either investor risk-aversion is lower or investor income and EM output is less correlated, then the spread correlation becomes indistinguishable from the log output correlation. The correlation is slightly higher in the first column's baseline setting because of the $\gamma^w \, \text{Cov}(D^i, y^w | \mathcal{Z})$ term which shows up in (19). Given positive cross-EM output correlation and positive correlation of EM output and investor income, the covariance between each countries expected default prospects and investor income will tend to move in the same direction; that movement will have more of a quantitative impact on equilibrium prices the larger is γ^w.

Setting $\gamma^w = 2$ makes time-zero equilibrium default probabilities and spreads higher, as well as increasing the standard deviation of spread changes. When investors have lower risk-aversion, they offer EM's a higher price of debt for any given desired level of borrowing, encouraging the EM to borrow more and resulting in a higher chance of default. The time-zero spread is higher because of the higher risk of default; but the risk-premium on the spread — the difference between the probability of default and the spread — is smaller. The standard deviation of spreads is higher because of the non-linearity of default probabilities. When a country begins in time-zero with higher default risk, there is more variability in default prospects between time-zero and time-one.

The failure to come close to matching the spread correlation in the data is what justifies calling the spread correlation a "puzzle." Despite the presence of risk-averse common investors, the correlation in spreads mostly tracks the correlation in output. In order to match the spread correlation, further ingredients must be added to the baseline model.

6.3 Matching the Correlation

I introduce, separately, two sources of additional time-one variability: shocks to investor risk-aversion and shocks to the costs of default. Examining them separately allows for a cleaner understanding of the mechanisms at play. In appendix B, I also show what happens when you allow for time-one shocks to the risk-free rate. It turns out that risk-free rate shocks do not change the spread correlation from the baseline model.

6.3.1 Price of Risk Movement

Many different "fixes" to asset pricing puzzles are ways of introducing cyclical investor risk-attitudes. Cochrane (2017) is a good survey: whether it be habits in consumption, long-run risks and recursive utility, idiosyncratic risk, or segmented financial markets, they all make the investor's SDF move more for a given movement in aggregate fundamentals than standard power utility would predict.

In order to generate movement in the price of risk, I introduce cyclical time-one shocks to investor risk-aversion γ^w. These shocks do not provide a micro-founded story like the just mentioned models, but they work with a similar mechanism, and leave open that the "true" model is some mixture of all of the above, rather than one specific story. The shocks take on the following form:

$$\gamma_1^w = \max\left\{12 - \varphi\left(k^w \frac{Y_1}{\sigma_Y} + \frac{w_1}{\sigma_w}\right),\ 0.5\right\} \tag{20}$$

Investor risk-aversion moves in proportion to the shock to each component of its income

Table 7: Stochastic Risk-Aversion

	B/Y (%)	P(Default)	Spread (%)	$\text{Cor}(\Delta S^1, \Delta S^2)$	$\text{sd}(\Delta S)$	$\text{Cor}(S, Z)$
Target	0.46	2.00	3.89	0.67	1.29	-0.30
Model	0.46	1.85	3.83	0.67	1.01	-0.77

process, normalized by their standard deviation, and weighted by the investor loading on each component. φ determines investor risk-aversion variability. As long as $\varphi > 0$, investor risk-aversion will be counter-cyclical. The max operator with 0.5 ensures that risk-aversion is positive.

Table 7 reports the results for what is otherwise the same baseline model, adding in time-one shocks to γ^w. $\varphi = 5.8$ is calibrated to match the cross-country spread correlation. The final column of the table adds an additional moment — the correlation between a country's spread and log output, which is -0.3 in my sample.

The presence of cyclical investor risk-aversion allows the model to match the correlation perfectly. Both countries spreads move up and down with investors desire to bear risk, generating the high correlation.

Even with variable risk-aversion, however, the model standard deviation of the change in the spread still undershoots the data. Part of the reason for this is the upward sloping term structure of spreads. Default probabilities and spreads are typically low at time-one, and changing investor risk-aversion can no longer affect default probabilities (it can at time-zero). Adding in longer-term debt — which could be represented by greater income variability between time-one and time-two — would significantly increase the standard deviation.

The moment the model really gets wrong is the correlation between spreads and output. Even though movements in investor risk-aversion are only weakly correlated with EM output (-0.3), spreads remain tightly linked to output. This is a common issue in default models. For example, in Morelli, Ottonello, and Perez (2022), spreads and output have a -0.84 correlation.

The standard deviation of γ^w with $\varphi = 5.8$ is 6.6. So, 95% of the time $\gamma^w \in [0.5, 25.2]$. This amount of risk-aversion movement may seem extreme, but is in line with, or even less than, popular asset pricing models. For example, in Campbell and Cochrane (1999), investor risk-aversion[8] moves between a value of 60 and in the hundreds over the business cycle.

[8] Measured as the second partial derivative of the value function with respect to individual wealth. See their discussion on p.243-44

Table 8: **Correlated Default Cost Shocks**

	B/Y (%)	P(Default)	Spread (%)	$\text{Cor}(\Delta S^1, \Delta S^2)$	sd(ΔS)	Cor(S, Z)
Target	0.46	2.00	3.89	0.67	1.29	−0.30
Model	0.47	1.79	3.91	0.67	1.57	−0.54

6.3.2 Quantity of Risk Movement

The other possible channel that could explain spread correlations is movements in the quantity of risk — cross-EM default prospects — that are not directly captured by standard fundamentals. Long-run risks to EM output in Bai et al. (2023) act as this sort of quantity of risk movement.

I propose quantity of risk movement stemming from variation in the *costs* of default, inspired by the model of Arellano, Bai, and Lizarazo (2017). In their model, countries benefit from defaulting "together" because they can jointly renegotiate the terms of their default and recovery. When multiple countries are in default, lenders outside option of failing to renegotiate is more costly, giving countries more bargaining power.

They provide a Nash-bargaining model of this process, which I abstract from here. Instead, consumption in case of default is now given by:

$$c_2^i = e^{z_2^i} - \phi_1 e^{\phi_3 (z_2^i + r_2^i)} \tag{21}$$

where the curvature parameter ϕ_3 is different from ϕ_2 and r_2^i is a stochastic default cost shock:

$$r_2^i = \rho_r r_1^i + \epsilon_{2,r} \text{ where } \epsilon_{2,r} \sim N(0, \sigma_s^2) \text{ and } r_1^i = \epsilon_{1,r} \tag{22}$$

The shock has mean zero and is independent of income shocks ϵ_Y, ϵ_x, but possibly correlated across countries, with $\text{Cor}(\epsilon_r^1, \epsilon_r^2) = \rho_s$. It follows an AR(1) process so that "news" about the time-two shock to default costs comes out at time-one. These shocks should be thought of as representing political or legal costs of default, connected across countries, but independent from output.

In order to restrict modeling degrees of freedom, I assume that ϕ_1 remains the same as before and that the variance of the shock to default costs is equal to the variance of output, $\sigma_r^2 = \sigma_z^2$.

With these restrictions, in order to match the spread correlation of 0.67 the default cost shocks must be both correlated and persistent. I calibrate that the cross-country correlation $\rho_s = 0.78$ and the auto-correlation $\rho_r = 0.8$. In appendix A.7 I provide full expressions for how default probabilities change.

Table 8 shows the results. As with risk-aversion shocks, the correlation of 0.67 is perfectly matched.

Uncertain default costs act just like uncertain income, pushing up the option value of default for the sovereign, and leading to investors demanding a greater risk-premium on the debt. The additional randomness increases the standard deviation of the change in the spread above its target value of 1.29 pp, but only mildly, to 1.57 pp. For comparison, the mean of country means (rather than the median) is 1.97 pp.

The cross-EM correlated default cost shocks are disconnected from country output, which reduces the correlation between spreads and output to -0.54, much lower than other model iterations. But even in the presence of these idiosyncratic shocks, the spread-output correlation is still almost double its value in the data.

6.4 Discussion

Independent sources of quantity of risk and price of risk movement are each able to match the cross-country correlation in spreads. Furthermore, the amount of investor risk-aversion variability needed to match the correlation is well within the bounds of risk-aversion variation in more micro-founded models. The correlation in default cost shocks needed to match the data seem implausibly high to me, but default costs remain a black box, hard to assess with data.

Examining these two possible sources of spread correlations separately helps clarify a few observations: 1) in reality, likely some combination of these two factors explain a significant amount of why spread correlations are much higher than fundamental correlation. 2) There remains a question of why the correlation between spreads and output is so low. The fact that movement in spreads is so poorly accounted for in the regressions of section 3.2 suggests that additional sources of variability, not tied to fundamentals, are needed to match the data. Bai et al. (2023) use long-run risks for exactly this reason. 3) More empirical work is desperately needed to discipline the various mechanisms at-play here. One potential advantage of using correlated default costs as a source of quantity of risk variation, as opposed to long-run risks, is that they should at least make predictions about realized default correlations.

7 Conclusion

Across worldwide emerging market economies, the spread on sovereign borrowing costs is highly correlated. The 0.67 median cross-country correlation in spreads far outstrips the correlation in country-level fundamentals like GDP (0.33) or debt-to-GDP (0.26). Empirically, I show that regressions of the spread on country-level and global variables only sharpens the disconnect between spreads and fundamentals. The R^2 in a regression of changes in the spread on a host of country-level variables is a mere 0.01. Global financial variables are much more capable of explaining spread movements than local variables, but the total amount of spread change variation they account for is still small (0.15 R^2). A new piece of evidence for the importance of global investors in driving spread movement is that (foreign currency) spreads are more sensitive to S&P 500 movement when foreign investors hold a greater fraction of a country's debt, or when a greater share of that country's debt is denominated in foreign currency.

To rationalize these empirical facts, I built a simple but flexible three-period two-country endogenous default model. The model shows that "baseline" sovereign default model features are insufficient to match high cross-country spread correlations. Instead, I show that two different mechanisms could account for high cross-country spread correlations. One is variability in investors risk-aversion driving movement in the price of risk. The other is variability in cross-country linked costs of defaulting. Both of these mechanisms are supported by the data, so a true account of high spread correlations would likely combine the two. However, existing data cannot precisely disambiguate each source of correlation's contribution, and both model variants still suffer from the problem of over-predicting the correlation between spreads and country output. Explaining high spread correlations with a mechanism that can be disciplined by the data, while avoiding overly high spread-output correlations, remains an open question.

References

Aguiar, Mark, and Gita Gopinath. 2006. "Defaultable Debt, Interest Rates and the Current Account." Journal of International Economics, Emerging Markets, 69 (1): 64–83.

Aguiar, Mark, Satyajit Chatterjee, Harold L. Cole, and Zachary Stangebye. 2016. "Quantitative Models of Sovereign Debt Crises." SSRN Scholarly Paper.

Al-Amine, Ramzy, and Tim Willems. 2023. "Investor Sentiment, Sovereign Debt Mispricing, and Economic Outcomes." The Economic Journal 133 (650): 613–36.

Arellano, Cristina. 2008. "Default Risk and Income Fluctuations in Emerging Economies." American Economic Review 98 (3): 690–712.

Arellano, Cristina, Yan Bai, and Sandra Lizarazo. 2017. "Sovereign Risk Contagion." Working Paper. Working Paper Series. National Bureau of Economic Research.

Bai, Yan, Kehoe, Patrick J., Lopez, Pierlauro, and Perri, Fabrizio. 2023. "World Financial Cycle." Work in progress, available on Yan Bai's personal website.

Bansal, Ravi, and Amir Yaron. 2004. "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles." The Journal of Finance 59 (4): 1481–1509.

Bauer, Michael D., and Eric T. Swanson. 2020. "A Reassessment of Monetary Policy Surprises and High-Frequency Identification." Working Paper. Working Paper Series. National Bureau of Economic Research.

Borri, Nicola, and Adrien Verdelhan. 2011. "Sovereign risk premia." In AFA 2010 Atlanta Meetings Paper.

Campbell, John Y., and John H. Cochrane. 1999. "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior." Journal of Political Economy 107 (2): 205–51.

Chen, Hui, Winston Wei Dou, and Leonid Kogan. 2022. "Measuring 'Dark Matter' in Asset Pricing Models." SSRN Scholarly Paper.

Cochrane, John H. 2011. "Presidential Address: Discount Rates." The Journal of Finance 66 (4): 1047–1108.

Cochrane, John H. 2017 "Macro-Finance." Review of Finance 21 (3): 945–85.

de Ferra, Sergio, and Enrico Malluci. 2022. "Sovereign Risk Matters: Endogenous Default Risk and the Time-Varying Volatility of Interest Rate Spreads." Journal of International Economics 134 (January): 103542.

de Ferra, Sergio, and Federica Romei. 2023. "Sovereign Default in a Monetary Union." Working Paper, available on the author's website.

Eaton, Jonathan, and Mark Gersovitz. 1981. "Debt with Potential Repudiation: Theoretical and Empirical Analysis." The Review of Economic Studies 48 (2): 289–309.

Epstein, Larry, Emmanuel Farhi, and Tomasz Stralezcki. 2014. "How Much Would You Pay To Resolve Long-Run Risk?" American Economic Review 104 (9): 2680–97.

Erce, Aitor, Enrico Mallucci, and Mattia Picarelli. 2022. "A Journey in the History of Sovereign Defaults on Domestic-Law Public Debt." International Finance Discussion Papers.

Fang, Xiang, Bryan Hardy, and Karen K. Lewis. 2022. "Who Holds Sovereign Debt and Why It Matters." Working Paper. Working Paper Series. National Bureau of Economic Research.

Gilchrist, Simon, Bin Wei, Vivian Z. Yue, and Egon Zakrajšek. 2022. "Sovereign Risk and Financial Risk." Journal of International Economics, NBER International Seminar on Macroeconomics 2021, 136 (May): 103603.

Guimaraes, Bernardo, and Lucas Tumkus. 2020. "On the Costs of Sovereign Default in Quantitative Models." Discussion Papers 2021, Centre for Macroeconomics (CFM).

Hamilton, James D. 2018. "Why You Should Never Use the Hodrick-Prescott Filter." The Review of Economics and Statistics 100 (5): 831–43.

Johri, Alok, Shahed Khan, and César Sosa-Padilla. 2022. "Interest Rate Uncertainty and Sovereign Default Risk." Journal of International Economics 139 (November): 103681.

Longstaff, Francis A., Jun Pan, Lasse H. Pedersen, and Kenneth J. Singleton. 2011. "How Sovereign Is Sovereign Credit Risk?" American Economic Journal: Macroeconomics 3 (2): 75–103.

Medas, Paulo, Tigran Poghosyan, Yizhi Xu, Juan Farah-Yacoub, and Kerstin Gerling. 2018. "Fiscal Crises." Journal of International Money and Finance 88 (November): 191–207.

Mehra, Rajnish, and Edward C. Prescott. 1985. "The equity premium: A puzzle." Journal of monetary Economics 15, no. 2: 145-161.

Miranda-Agrippino, Silvia, and Hélène Rey. 2020. "U.S. Monetary Policy and the Global Financial Cycle." The Review of Economic Studies 87 (6): 2754–76.

Morelli, Juan M., Pablo Ottonello, and Diego J. Perez. 2022. "Global Banks and Systemic Debt Crises." Econometrica 90 (2): 749–98.

Onen, Mert, Hyun Song Shin, and Goetz von Peter. 2023. "Overcoming Original Sin: Insights from a New Dataset," BIS Working Papers.

Park, Jungjae. 2014. "Contagion of Sovereign Default Risk: the Role of Two Financial Frictions." Available at SSRN 2530817.

Rey, Hélène. 2013. "Dilemma not Trilemma: The Global Financial Cycle and Monetary Policy Independence", Federal Reserve Bank of Kansas City Economic Policy Symposium.

Shin, Hyun Song. 2012. "Global Banking Glut and Loan Risk Premium." IMF Economic Review 60 (2): 155–92.

Shiller, Robert J. 1981. "Do Stock Prices Move Too Much to Be Justified by Subsequent Changes in Dividends?" The American Economic Review 71 (3): 421–36.

Tourre, Fabrice. 2017. "A Macro-Finance Approach to Sovereign Debt Spreads and Returns." Society for Economic Dynamics, 2017 Meeting Papers 13.

Vasicek, Oldrich. 2002. "The Distribution of Loan Portfolio Value, Risk." Available via the Internet: www.moodyskmv.com/conf04/pdf/papers/dist_loan_port_val.pdf.

A Appendix

A.1 General Correlation Expression

To derive (6) I begin by writing out $\mathbb{E}\big((q^1|Z)(q^2|Z)\big)$:

$$\begin{aligned}\mathbb{E}\big(q^1q^2\big) =& \,\mathbb{E}\big(\,\mathbb{E}\big(mD^1|Z\big)\,\mathbb{E}\big(mD^2|Z\big)\big) - \,\mathbb{E}\big(\,\mathbb{E}\big(mD^1|Z\big)\,\mathbb{E}(m|Z)\big) - \,\mathbb{E}\big(\,\mathbb{E}(m|Z)\,\mathbb{E}\big(mD^2|Z\big)\big) + \,\mathbb{E}\Big(\,\mathbb{E}(m|Z)^2\Big) \\ =& \,\mathrm{Cov}\big(\,\mathbb{E}\big(mD^1|Z\big),\,\mathbb{E}\big(mD^2|Z\big)\big) + \,\mathbb{E}\big(mD^1\big)\,\mathbb{E}\big(mD^2\big) - \,\mathrm{Cov}\big(\,\mathbb{E}\big(mD^1|Z\big),\,\mathbb{E}(m|Z)\big) - \\ & \mathbb{E}\big(mD^1\big)\,\mathbb{E}(m) - \,\mathrm{Cov}\big(\,\mathbb{E}\big(mD^2|Z\big),\,\mathbb{E}(m|Z)\big) - \,\mathbb{E}\big(mD^2\big)\,\mathbb{E}(m) + \,\mathrm{Var}(\,\mathbb{E}(m|Z)) + \,\mathbb{E}(m)^2\end{aligned}$$

When you subtract off $\mathbb{E}\big(q^1\big)\,\mathbb{E}\big(q^2\big)$ the first-order terms disappear:

$$\begin{aligned}\mathrm{Cov}\big(q^1, q^2\big) =& \,\mathrm{Cov}\big(\,\mathbb{E}\big(mD^1|Z\big),\,\mathbb{E}\big(mD^2|Z\big)\big) - \,\mathrm{Cov}\big(\,\mathbb{E}\big(mD^1|Z\big),\,\mathbb{E}(m|Z)\big) - \\ & \mathrm{Cov}\big(\,\mathbb{E}\big(mD^2|Z\big),\,\mathbb{E}(m|Z)\big) + \,\mathrm{Var}(\,\mathbb{E}(m|Z))\end{aligned}$$

Assuming that the variance of each country's debt price is the same, the denominator of the correlation expression is:

$$\mathrm{Var}\big(q^1\big) = \,\mathrm{Var}\big(\,\mathbb{E}\big(mD^1|Z\big)\big) - 2\,\mathrm{Cov}\big(\,\mathbb{E}\big(mD^1|Z\big),\,\mathbb{E}(m|Z)\big) + \,\mathrm{Var}(\,\mathbb{E}(m|Z))$$

To get (6), assume that $\mathrm{Cov}\big(\,\mathbb{E}\big(mD^1|Z\big),\,\mathbb{E}(m|Z)\big) = \,\mathrm{Cov}\big(\,\mathbb{E}\big(mD^2|Z\big),\,\mathbb{E}(m|Z)\big)$ and write things in terms of D^i.

A.2 Default probability derivation

The sovereign is indifferent between defaulting and repaying when $e^{z_2^i} - b^i = (1 - \phi_1 e^{\phi_2 z_2^i})e^{z_2^i}$, which results in a time zero default probability p^i given by

(13).

$$
\begin{aligned}
& e^{z_2^i} - b^i = (1 - \phi_1 e^{\phi_2 z_2^i}) e^{z_2^i} \\
\implies & b^i = \phi_1 e^{z_2^i (1+\phi_2)} \\
& \ln\left(\frac{b^i}{\phi_1}\right) = z_2^i (1 + \phi_2) \\
\implies & D^i = 1 \iff z_2^i < \frac{\ln\left(\frac{b^i}{\phi_1}\right)}{1 + \phi_2} \\
\implies & \mathbb{P}\big(D^i = 1\big) = \Phi\left(\frac{\ln\left(\frac{b^i}{\phi_1}\right)}{\sigma_z (1 + \phi_2)}\right)
\end{aligned}
$$

After the shock at time-one, default probabilities are given by:

$$
p^i = \Phi\left(\frac{\ln\left(\frac{b^i}{\phi_1}\right) - \mu_z}{\sigma_z'(1 + \phi_2)}\right)
$$

Where $\mu_z = \mathbb{E}\big(z_2^i | z_1^i\big)$ and σ_z' is the one-quarter long standard deviation of income.

A.3 Derivative of Asset Demand with respect to default probability

In the time zero equilibrium asset demand condition I substitute in that $\mathbb{E}\big(D^1\big) = p^1$, $\text{Var}\big(D^1\big) = p^1(1 - p^1)$, and that $\text{Cov}\big(D^1, y^w\big) = \mathbb{E}\big(D^1 y^w\big) - \mathbb{E}\big(D^1\big)\,\mathbb{E}(y^w)$. I also let $\mathbb{E}\big(e^{z_2^w} | D^1 = 1\big) = \mathbb{E}_{Z|D}$, and $\mathbb{E}\big(e^{z_2^w}\big) = \mathbb{E}_Z$:

$$
a_0^1 = \frac{q_0^1 \left(1 - p^1 - (1 + r_f) q_0^1\right) + \gamma^w q^1 p^1 (\mathbb{E}_{Z|D} - \mathbb{E}_Z)}{\gamma^w \left(p^1 - (p^1)^2 + \Phi_2(\Phi^{-1}(p^1), \Phi^{-1}(p^2); \rho_D) - p^1 p^2\right)} \tag{23}
$$

$\Phi_2(.,.;\rho_D)$ is the bivariate standard normal CDF with correlation ρ_D given by the correlation between default probabilities, which solely depends on the correlation between z_2^1 and z_2^2. This expression is derived in Vasicek (2002) and Shin (2012). It represents the probability that both countries default, which is equivalent to $\text{E}[D^1 D^2]$. To further simplify things, I refer to this expression as Φ_2, I denote the numerator as A, and the denominator as B.

The derivative with respect to p^1 depends on the derivative of Φ_2 with respect to p^1.

That derivative is equal to:[9]

$$\frac{\partial \Phi_2(.)}{\partial \Phi^{-1}(p^1)} \frac{\partial \Phi^{-1}(p^1)}{\partial p^1} = \Phi\left(\frac{\Phi^{-1}(p^2) - \rho_D \Phi^{-1}(p^1)}{\sqrt{1-\rho_D^2}}\right) \equiv \Phi_{11} \tag{24}$$

I will denote this expression with Φ_{11}. Now the partial with respect to p^1 is:

$$\frac{\partial a_0^1}{\partial p^1} = \frac{\left(\gamma^w(\mathbb{E}_{Z|D} - \mathbb{E}_Z) - q_0^1\right) B - \gamma^w(1 - 2p^1 + \Phi_{11} - p^2)A}{B^2} \tag{25}$$

The sign of the numerator can be analyzed as follows:

$$\underbrace{\left(\gamma^w(\mathbb{E}_{Z|D} - \mathbb{E}_Z) - q_0^1\right)}_{<0} \underbrace{B}_{>0} - \underbrace{\gamma^w(1 - 2p^1 + \Phi_{11} - p^2)}_{>0 \text{ if } p^1<0.5} \underbrace{A}_{>0} \tag{26}$$

The sign of the term multiplying B is determined by the fact that $\mathbb{E}_{Z|D} < \mathbb{E}_Z$ when output is positively correlated across EM and investor income. The sign of B is determined by:

$$p^1 - (p^1)^2 > 0 \text{ and } \Phi_2 - p^1 p^2 > 0 \implies B > 0 \tag{27}$$

Since the bivariate CDF represents the probability of both countries defaulting, it must be greater than $p^1 p^2$, as long as defaults are positively correlated (which they are).

For A and the term multiplying A I verify numerically, using my calibration, that across all values of equilibrium debt (and all default correlations) the former is always positive while the latter is negative whenever $p^1 < 0.5$.

Therefore, the entire numerator is negative and:

$$\frac{\partial a_0^1}{\partial p^1} < 0 \tag{28}$$

Asset demand is decreasing in default probability.

[9] Thanks to Stack Overflow contributors "josliber" and "Henry" for this derivation. The partial derivative of the bi-variate CDF is derived here: https://stats.stackexchange.com/questions/71976/partial-derivative-of-bivariate-normal-cdf-and-pdf. The partial derivative of the inverse CDF with respect to its argument is derived here: https://math.stackexchange.com/questions/910355/derivative-of-the-inverse-cumulative-distribution-function-for-the-standard-norm

A.4 Other Asset Demand Comparative Statics

- It is clear from (23) that demand is decreasing in default correlation, since increasing default correlation increases Φ_2 and nothing else.

- The derivative with respect to country two default probability, holding everything else fixed, is positive.

$$\frac{\partial a_0^1}{\partial p^2} = \frac{-\gamma^w(\Phi_{12} - p^1)A}{B^2} \tag{29}$$

  Where Φ_{12} is the same as Φ_{11} in (24) except with respect to p^2.

  This expression is positive whenever:

$$\begin{aligned} &\Phi_{12} - p^1 < 0 \\ \implies &\frac{\Phi^{-1}(p^2) - \rho_D \Phi^{-1}(p^1)}{\sqrt{1-\rho_D^2}} < \Phi^{-1}(p^1) \\ &\Phi^{-1}(p^1)(1-\rho_D) < \Phi^{-1}(p^1)\sqrt{1-\rho_D^2} \\ \implies &1 - \rho_D < \sqrt{1-\rho_D^2} \end{aligned}$$

  When $\rho_D < 1$ this must be true.

- The derivative with respect to $\mathrm{Cov}(D^1, e^{z_2^w})$ is positive (more covariance implies more insurance), since it appears only in the numerator multiplied by $\gamma^w q^1 p^1$, all of which are greater than zero.

- The derivative with respect to risk-aversion is negative, as can be seen from numerator of $\frac{\partial a_0^1}{\partial \gamma^w}$:

$$\underbrace{q_0^1 \mathrm{Cov}(D^1, e^{z_2^w})}_{<0} \underbrace{B}_{>0} - \underbrace{(1 - (p^1)^2 + \Phi_2 - p^1 p^2)}_{>0} \underbrace{A}_{>0} < 0 \tag{30}$$

- The derivative with respect to the risk-free rate is straightforwardly negative, since it only is in the numerator and is multiplied by $-(q_0^1)^2$

A.5 Solving the Model

To solve the model at time-zero, I loop over possible borrowing values b^i, calculate prices q^i for each b^i, and keep track of the sovereign's maximum value of $u(c_1) + \beta\,\mathbb{E}(u(c_2))$.

The expectation of period-2 sovereign utility is calculated using Monte-Carlo integration:

$$\mathbb{E}(u(c_2)) = \int \max\Big\{u\left(e^{z_2} - b\right), u\left(e^{z_2}\left(1 - \phi_1 e^{\phi_2 z_2}\right)\right)\Big\} dz$$

In order to calculate prices, I use the bi-variate CDF in (23) for $\text{Cov}\big(D^1, D^2\big)$ and I use a simulation for $\text{Cov}\big(D^i, y^w\big)$. To compute the bi-variate CDF requires default correlation ρ_D at time-zero, which is different from the one-quarter ahead correlation. It is given by:

$$\text{Cor}\big(z_2^1, z_2^2\big) = \frac{k_{EM}^2 \sigma_Y^2 \left(\rho_Y + 1\right)}{k_{EM}^2 \sigma_Y^2 \left(\rho_Y + 1\right) + \sigma_x^2 \left(\rho_x + 1\right)}$$

The simulation for $\text{Cov}\big(D^i, y^w\big)$ takes advantage of the fact that $\text{Cov}\big(D^i, y^w\big) = \mathbb{E}\big(D^i y^w\big) - \mathbb{E}\big(D^i\big)\,\mathbb{E}(y^w) = p^i\left(\mathbb{E}\big(y^w | D^i = 1\big) - \mathbb{E}(y^w)\right)$. For a given level of b^i, I simulate the model over 1 million draws from time-zero, time-one, and time-two output sequences. I keep track of default frequencies and then compute the sample average of investor income conditional upon default minus the full sample average of investor income. Ideally, I would prove that the sample average of investor income conditional upon default converges to $\mathbb{E}\big(y^w | D^i = 1\big)$. I have not done that, but I have verified that when using 1 million draws the sample average is stable over multiple runs.

To solve the model at time-one, all I need to do is recompute prices conditional on a draw from the income distribution. $\mathbb{E}\big(D^i | \mathcal{Z}\big)$, $\text{Var}\big(D^i | \mathcal{Z}\big)$, $\text{Cov}\big(D^1, D^2 | \mathcal{Z}\big)$ can all be computed analytically, while $\mathbb{E}\big(y^w | D^i = 1, \mathcal{Z}\big)$ is again determined numerically. The time-one model moments I report are based off of 100,000 draws of time-one income shocks.

A.6 Calibration Details

All of the following correlation and variance expressions are the *one-quarter* ahead correlations and variances. They also depend on my assumption that $\sigma_Y = \sigma_x$.

- k^{EM} is set so that $\text{Cor}\big(z_{EM}^1, z_{EM}^2\big) = \frac{k_{EM}^2}{k_{EM}^2 + 1} = 0.325$
- I set $\sigma_Y = \sigma_x$ using $\text{Var}(z_{EM}) = \sigma_Y^2 (k_{EM}^2 + 1) = .0285^2$
- k_w, σ_w and ρ_w are determined numerically to match $\text{Cor}(z_{EM}, z_w) = 0.435$, $\sigma_{US}^2 = .0195$ and a US output auto-correlation of 0.72. The parameters are such that the time-one moments match the correlations and standard deviations, while an extended sequence of investor income would match the auto-correlation.

- ρ_Y, ρ_x directly set to 0.68 to match EM output auto-correlation

- Fang, Hardy, and Lewis (2022) list six investor types: domestic banks, domestic non-banks, domestic central banks, foreign banks, foreign non-banks, and foreign central banks. For each investor type, *among those that hold EM sovereign debt*, the share of their total wealth in EM sovereign debt is given in table 3 of the paper. Table 1 gives the average share of EM sovereign debt held by investor type. I multiply each investor types share of wealth in EM debt by their share of EM sovereign debt to construct a "representative" debt investor. In Morelli, Ottonello, and Perez (2022) they have their down data on global bank specific holdings of EM debt, and calibrate their model to match the fact that 10% of global banks risky assets are in EM sovereign debt.

A.7 Stochastic Default Costs

Consider default costs as given in (21). $\text{Var}\big(r^i\big)$ is assumed equal to $\text{Var}\big(z^i\big)$, so $\sigma_r^2 = \sigma_z^2$.

The default probability is now determined by $\mathbb{P}\Big(\phi_3(z_2^i + r_2^i) < \ln\Big(\frac{b^i}{\phi_1}\Big)\Big)$, which means:

$$p^i = \Phi\left(\frac{\ln\left(\frac{b^i}{\phi_1}\right) - \mu_{z+r}}{\phi_3\sqrt{\sigma_z^2 + s_r^2}}\right)$$

Where I use $\mu_{z+r} = \mathbb{E}\big(z_2^i + r_2^i\big)$, which is zero at time-zero and at time one equals:

$$\mathbb{E}\big(z^i + r^i|\mathcal{Z}\big) = k_{EM}\rho_Y Y_1 + \rho_x x_1^i + \rho_r \epsilon_{1,r}$$

$\text{Cov}\big(D^1, D^2\big)$, which is need to calculate prices, is simply the probability that both countries default minus their individual default probabilities. The expression for joint default is a bi-variate CDF, as in (23), with mean arguments corresponding to individual default probabilities and the time-zero correlation coefficient given by:

$$\text{Cor}\big(z^1 + r^1, z^2 + r^2\big) = \frac{\sigma_Y^2\left(\rho_Y^2 + 1\right)\left(k_{EM}\right)^2 + \rho_s \sigma_r^2(\rho_r^2 + 1)}{\sigma_z^2 + \sigma_r^2}$$

The time-one expressions conditional on the shock drop the ρ^2 terms that are auto-correlation coefficients.

B Stochastic Risk-Free Rate

There are two ways to introduce a stochastic risk-free rate into the model. The easier way is to have time-one shocks to the risk-free rate, akin to the time one shocks to investor risk-aversion. These shocks are unanticipated by investors, so it does not affect their time zero problem, but modifies the way prices update at time one. However, this method may be unsatisfactory insofar as at time zero investors are treating the risk-free rate as if it is a two-period long *risk-free* rate. This is unlike shocks to risk-aversion, where there is no sense in which investors are planning around their risk-aversion being unchanging.

To introduce risk-free rate shocks at time one when investors know that such shocks are possible, the investor problem must be modified so that the expectation and variance of period two wealth are as follows. I now use subscripts to denote the two countries prices q_1, q_2, default probabilities D_1, D_2, and demands a_1, a_2 so that squared terms are unambiguous. All variables are time-zero variables.

$$\mathbb{E}(w) = (1 - a_1 - a_2)(1 + \mathbb{E}(r_f)) + \frac{a_1}{q_1}(1 - \mathbb{E}(D_1)) + \frac{a_2}{q_2}(1 - \mathbb{E}(D_2)) + \mathbb{E}\big(e^{Z_w}\big) \tag{31}$$

$$\begin{aligned}
\mathrm{Var}(W) = &\left(\frac{a_1}{q_1}\right)^2 \mathrm{Var}(D_1) + \left(\frac{a_2}{q_2}\right)^2 \mathrm{Var}(D_2) + (1 - a_1 - a_2)^2 \,\mathrm{Var}(r_f) + \mathrm{Var}\big(e^{Z_w}\big) + \\
2&\Bigg(\frac{a_1 a_2}{q_1 q_2} \mathrm{Cov}(D_1, D_2) + \frac{a_1^2}{q_1} \mathrm{Cov}(D_1, r_f) - \frac{a_1}{q_1}\mathrm{Cov}(D_1, e^{Z_w}) - \frac{a_2}{q_2} \mathrm{Cov}\big(D_2, e^{Z_w}\big) + \frac{a_2^2}{q_2} \mathrm{Cov}(D_2, r_f) \\
&- a_1 \,\mathrm{Cov}\big(r_f, e^{Z_w}\big) - a_2 \,\mathrm{Cov}\big(r_f, e^{Z_w}\big) + \mathrm{Cov}\big(r_f, e^{Z_w}\big) + \frac{a_1 a_2}{q_2} \mathrm{Cov}(D_2, r_f) + \frac{a_1 a_2}{q_1} \mathrm{Cov}(D_1, r_f) \\
&- \frac{a_1}{q_1} \mathrm{Cov}(r_f, D_1) - \frac{a_2}{q_2} \mathrm{Cov}(r_f, D_2)\Bigg)
\end{aligned} \tag{32}$$

Re-arranging the FOC with respect to a_1 gives the following, which allows for isolating a_1:

$$\begin{aligned}
\gamma a_1 &\left(\frac{\mathrm{Var}(D_1)}{q_1^2} + 2\frac{\mathrm{Cov}(D_1, r_f)}{q_1} + \mathrm{Var}(r_f)\right) = \frac{1 - \mathbb{E}(D_1)}{q_1} - (1 + \mathbb{E}(r_f)) - \gamma\Bigg(\frac{a_2}{q_1 q_2} \mathrm{Cov}(D_1, D_2) - \\
&\frac{1}{q_1} \mathrm{Cov}\big(D_1, e^{Z_w}\big) - \mathrm{Cov}\big(r_f, e^{Z_w}\big) + \frac{a_2}{q_1} \mathrm{Cov}(D_1, r_f) + \frac{a_2}{q_2} \mathrm{Cov}(D_2, r_f) - (1 - a_2) \,\mathrm{Var}(r_f) - \frac{1}{q_1} \mathrm{Cov}(D_1, r_f)\Bigg)
\end{aligned}$$

I calibrate the model so that:

- $\mathrm{Cov}\big(D^1, r_f\big) = \mathrm{Cov}\big(D^2, r_f\big) = .000054$ is exogenously set to match the observed

Table 9: Stochastic Risk-Free Rate

	B/Y (%)	P(Default)	Spread (%)	$\text{Cor}(\Delta S^1, \Delta S^2)$	$\text{sd}(\Delta S)$	$\text{Cor}(S, Z)$
Target	0.46	2.00	3.89	0.67	1.29	-0.30
Baseline Model	0.46	1.85	3.83	0.36	0.65	-0.90
Stochastic r_f	0.46	1.88	3.59	0.35	0.66	-0.90

correlation between the Medas et al. (2018) default data set and the Wu-Xia shadow fed funds rate series.

- Johri et al. (2022) fit a stochastic volatility model of the world interest rate with a 97 basis point annualized standard deviation. This corresponds to a one-quarter ahead quarterly standard deviation of 0.00485 and a time zero (two-quarter ahead) standard deviation of 0.00686.

- I set $\text{Cov}(r_f, y^w) = .00016$ to match the covariance between the shadow fed funds rate and log filtered GDP in my data.

With everything else from the baseline calibration the same as in section 6.1, the model produces the moments shown in Table 9. I report the baseline model moments as well for ease of comparison.

The two models are almost indistinguishable, and there is no significant difference in the cross-country spread correlations the stochastic risk-free rate model produces. The reason is that while prices move with the risk-free rate, the spread is measured with respect to the risk-free rate, so even though price movement increases substantially, spread movement does not.